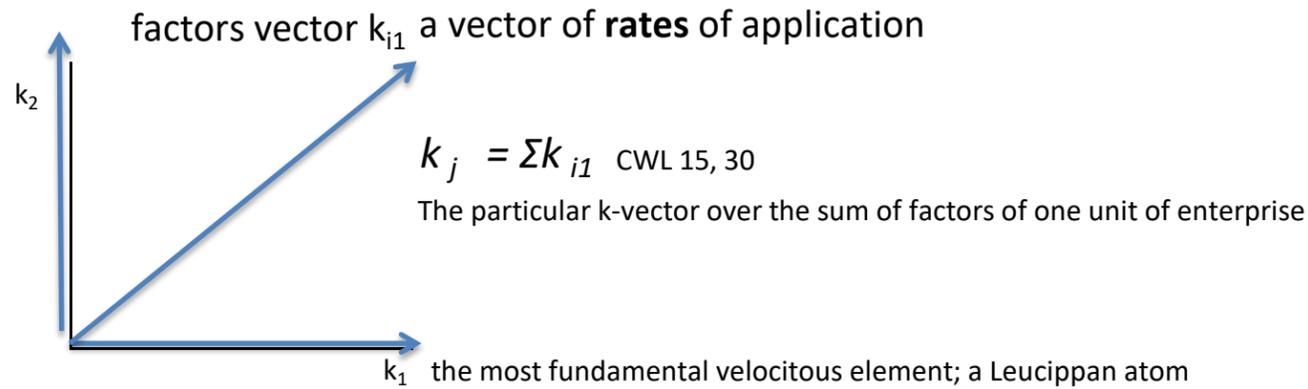


The Einstein-Hilbert Context and Systematics of the Economic Process

The Formal Similarity of the Relativities of Space-and-Time and Price-and-Quantity Instantaneous Rates

Section I. Aggregation of microeconomic into macroeconomic Quantities, Prices, and Values

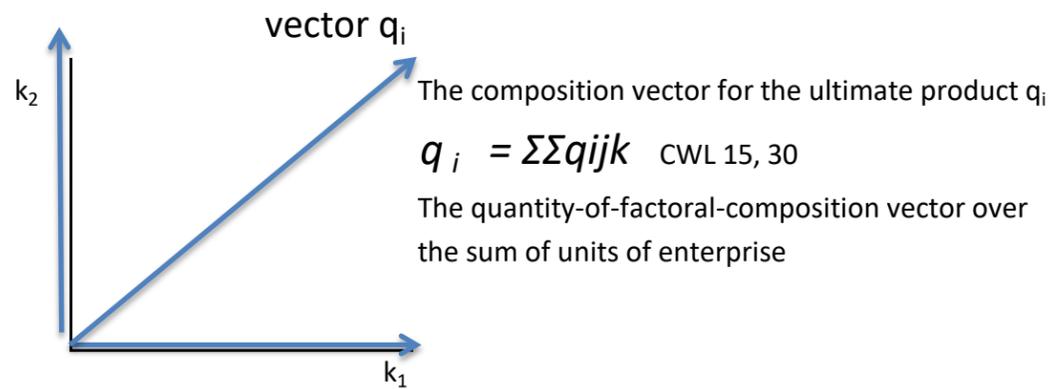
The atoms of rates of application of 1) factors of production and 2) correlated compensatory outlays become Gross Domestic Functional Flows



Similar Form(s)

vector p_j of pricing in an interval

the most fundamental element of outlay- income; a Leucippan atom

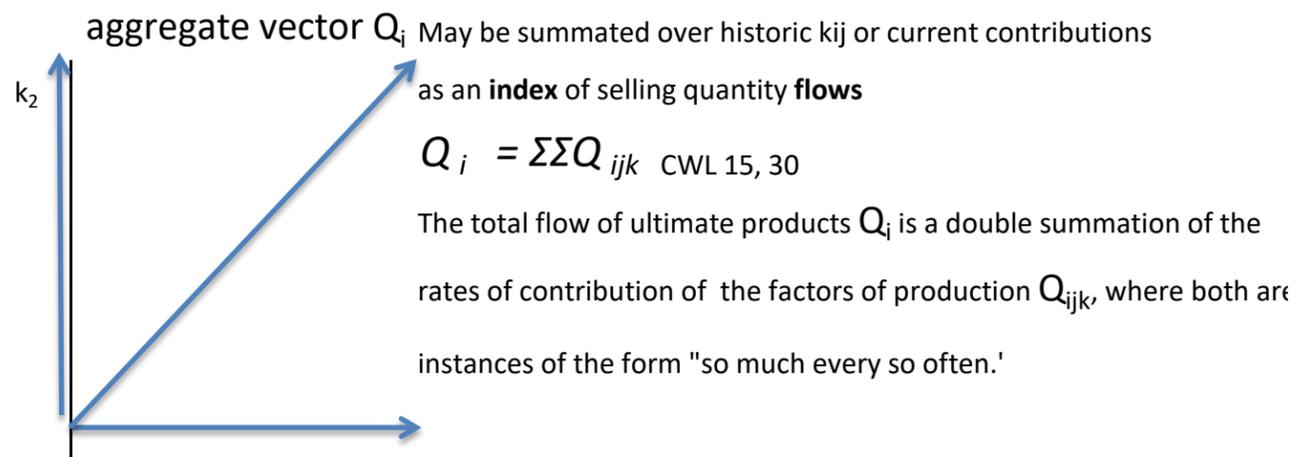


Similar Form(s)

vector p_i for factor cost price

The ultimate outlays for product q_i

The quantity-of-monetary-composition vector over the sum of units of enterprise



Similar Aggregate Form(s)

aggregate vector P' a selling pricing **index**

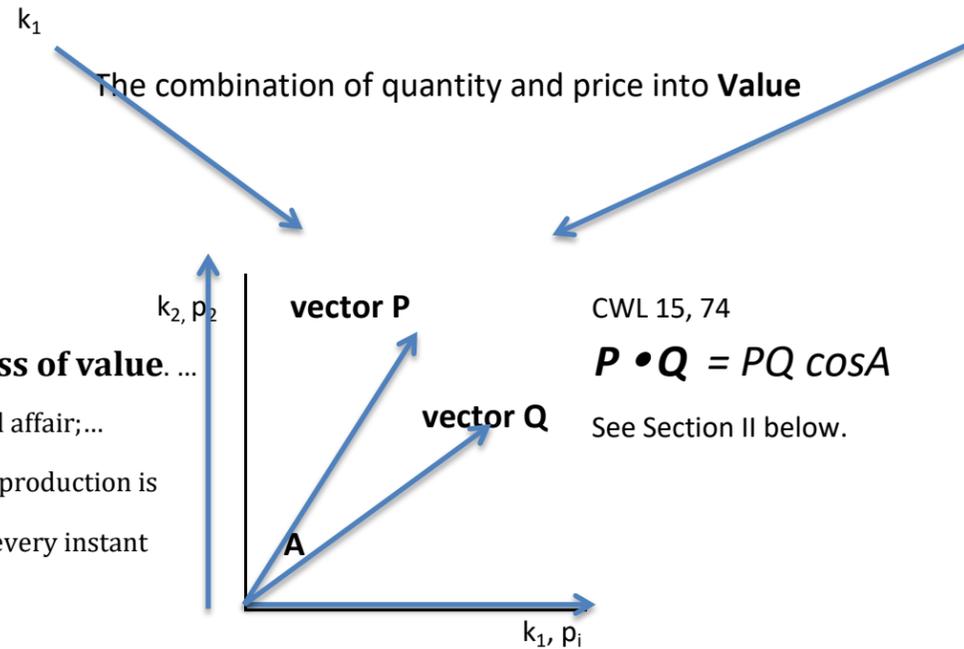
vector $\alpha'Q'$ | Basic costs of basic goods

vector $\alpha''Q''$ | Basic costs of surplus goods

vector $\alpha'''K_i'''$ | Purely expansionary

vector $\alpha''''K_i''''$ | Repair and maintenance to self

vector P' | Total basic Expenditures



The productive process (is) **a process of value**. ...
 ... production is not a merely technical affair;...
 intrinsically it is an economic affair, ... production is
 for sale, production in view of and at every instant
 adapted to payments. [CWL 21, 114]

CWL 15, 74
 $P \cdot Q = PQ \cos A$
 See Section II below.

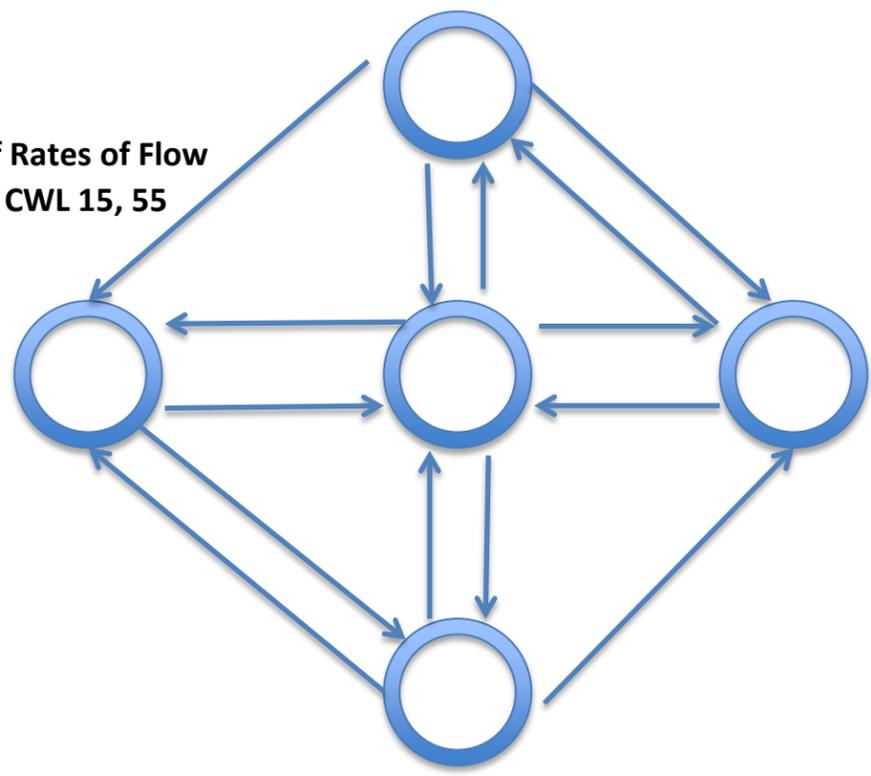
vector Π'' | Total investment expenditures

Similar Aggregate Equations
 $\Pi'' \cdot K'' = \Pi'' K'' \cos A$
 $p' \cdot a' Q' = (p')(a' Q') \cos A$
 $(\pi'') \cdot (a'' K'') = (\pi'')(a'' K'') \cos A$

The Summary of Flows of the Economic Process (prescinding the granting of "false" credit)

Normative Gross Domestic Functional Flows: $P'Q' + \Pi''K'' = p'a'Q'_{\text{Basic circuit cost}} + p''a''Q''_{\text{Basic R\&M}} + \pi''\alpha''K''_{\text{Pure expansionary surplus}} + \pi''\alpha''K''_{\text{Surplus R\&M to self}}$
 See <http://functionalmacroeconomics.com/table-of-contents/buildup-of-formulae/>

See Figure 14-1 Diagram of Rates of Flow
 CWL 15, 55



Section II. An exercise in the geometry of differing functional flows

An Advance from factoral elements to vectors, to the invariant algebra and trigonometry of the relativity of price and quantity.

Note that, as $ds^2 = dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2$ is the basic trigonometric form of Special Relativity, so the dot product, $P \bullet Q = PQ \cos A$ of vectoral price patterns and vectoral quantity patterns is the basic trigonometric form of price-quantity flows, whether of a) consumption Expenditures or Costs or b) Investment Expenditures or Pure Surplus Incomes.

Against the background of Special Relativity's constant velocity of light necessitating combinations of dilation of time and contraction of length,

we consider the possibility of a given economic velocity's (flow's) determining tradeoffs between its two mutually determining constituents, price and quantity.

"The question is, How much does the increment in the rate of payment DZ result from price increments dp , and how much does it result from quantity increments dq ?" [CWL 15, 108]

↓ This invariant trigonometric intelligibility comparing different flows applies to contemporaneous flows and to successive flows.

$$Z = \sum p_i q_i \quad [(13) \text{ for expenditures}] \quad [\text{CWL 15, 107-109}]$$

$$Z + DZ = \sum (p_i q_i + p_i dq_i + q_i dp_i + dp_i dq_i) \quad [(14) \text{ for expenditures}] \quad [\text{CWL 15, 107-109}]$$

$$\text{Therefore, } Z = \sum (p_i)(aq_i) \quad [\text{for "cost" as defined}]$$

$$Z + DZ = \sum (p_i aq_i + p_i daq_i + aq_i dp_i + dp_i daq_i) \quad [\text{for "cost" as defined}]$$

$$Z = \sum p_i q_i = \mathbf{P} \cdot \mathbf{Q} = PQ \cos A \quad [(17) \text{ for expenditures}] \quad [\text{CWL 15, 107-109}]$$

$$Z + DZ = (P+dP)(Q+dQ) \cos(A+dA) \quad [(20) \text{ for expenditures}] \quad [\text{CWL 15, 107-109}]$$

$$\text{Therefore, } \sum (p_i q_i + p_i dq_i + q_i dp_i + dp_i dq_i) = (P+dP)(Q+dQ) \cos(A+dA) \quad (14) \text{ and } (20)$$

$$DZ = PQ[(dP/P + dQ/Q + dPdQ/PQ) \cos(A+dA) - 2 \sin(dA/2) \sin(A+dA/2)]$$

[CWL 15, 107-109]; [VNR Encyclopedia, 1977. p. 234]

In sum, a special relativistic perspective in Functional Macroeconomic Dynamics: differing volumes of particular functional flows possess an algebraic and trigonometric intelligibility.

Similar forms

$$(\mathbf{p}') \bullet (\mathbf{a}'\mathbf{q}') = (p')(a'q') \cos A$$

Section III. Relativities in Macroeconomics and Physics

Macroeconomics

A functional flow has price and quantity components subject to "dilation" and "contraction."

By Lonergan's postulate of the normative reciprocity - symbolized by an equals sign - of a) basic expenditures (velocitous flows) and basic costs (velocitous flows), their separate

pretio-quantital flows will be subject to dilation and contraction of price vector and quantity

vector within to preserve the equality of flows on both sides of the equals sign

so as to avoid distortion of the process. Thus, price and quantity are relativistically **redefined**.

Special Relativity: redefinition of price and quantity within pretio-quantitality

(Vectorial Intelligibilities invariant across all economic systems)

↓ (Valued flow) = (valued flow) + (valued flow)

↓ Three functional flows are mutually determining and determined.

Physics

In Einstein's Special Relativity, the postulates of 1) invariance of the expression of principles and laws under inertial transformations, and 2) the constancy of the speed of light to observers under inertial transformations, require that Newton's and Galileo's 3-d absolute space and 1-d absolute time must be **redefined** as a 4-d spacetime.



Special Relativity: a relativistic redefinition of space and time within spacetime

(Algebraic & trigonometric invariants in all inertial reference systems)

↓ Speed of light same for all observers under inertial transformations

$$[P'Q'] = [(p'a'Q') + (p''a''Q'')] \text{ [CWL 15, 157-58]}$$

↓ Prices and quantities redefined as relativistic w

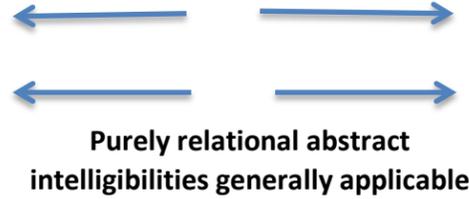
$$P'/p' = a' + a''p''Q''/p'Q' \text{ See footnotes 1 and 10}$$

$$P'/p' = a' + a''R, \text{ or}$$

$$J = a' + a''R$$

↓ Differentials

$$dJ = da' + a''dR + Rda'' \text{ See footnotes 1 and 10}$$



General Relativity

Loneragan's **postulate** and **principle** that the **process** of demand **expenditures** flows are reciprocal to, and not to be distinguished from, the **process** of supply-costs flows suggest his appreciation of Einstein's 1) principle of equivalence and 2) his equating the **process** of spacetime curvature with that **process** which **conserves energy and momentum**

Basic Expenditures are normatively reciprocal to the receipt of Basic Incomes

Each price and quantity index vector is a function of all the other index vectors

Let P'Q', p'a'Q', and p''a''Q'' be considered tensor forms.

↓ We have tensor equations; the left side tells the right how to act, and vice versa

$$P'Q' = p'a'Q' \text{ Basic outlays for basic income} + p''a''Q'' \text{ Surplus outlays for basic income}$$

$$\Pi''K'' = \pi''\alpha''K'' \text{ Purely expansionary outlays} + \pi''\alpha''K'' \text{ R\&M outlays to self}$$

↓ Gross Domestic Functional Flows

$$GDFF = P'Q' + \Pi''K''$$

$$k_n [f'_n(t-a) - B_n] = f''_{n-1}(t) - A_{n-1}$$

Applicable as a statement of both current relations

and as a statement suggesting how the process will evolve.

$$c_1 = c_2$$

↓ Redefinition and relativity of terms Footnote 15

$$\Delta t = \Delta t_0 / \sqrt{1 - v^2/c^2} \quad \text{Time dilation}$$

$$\Delta t_0 = (\Delta t) \sqrt{1 - v^2/c^2}$$

$$L = L_0 \sqrt{1 - v^2/c^2} \quad \text{Length contraction}$$

$$p = mv / \sqrt{1 - v^2/c^2} \quad \text{momentum}$$

$$KE = mc^2 (1/\sqrt{1 - v^2/c^2} - 1) \quad \text{momentum kinetic energy}$$

$$ds^2 = x_1^2 + x_2^2 + x_3^2 + ict^2$$

General Relativity

By Einstein's **principle of equivalence**: a gravitational force is **indistinguishable** from an acceleration of the system of reference.

By the **postulate of invariance** the principles and laws are **invariant** under any transformations.

Energy-Momentum is reciprocal to the curvature of spacetime

Ten tensorial equations linking twenty quantities

The curvature tells matter how to move, and

The distribution of matter determines the curvature of spacetime

$$G_{ab} = 8\pi T_{ab}$$

$$R_{ab} - .5g_{ab} R = kT_{ab}$$

$$ds^2 = \sum g_{ij} dx_i dx_j \quad [i, j = 1, 2, \dots, n]$$



Footnote 1: Prices and quantities must be defined relativistically within the principles and laws of functional flows. They are not given absolutes.

One might be reminded here of a parallel in hydrodynamics: if what is at issue is a general specification of the dynamics of free water waves, a premature introduction of general boundary conditions or worse, specific channel conditions, botches the analytic possibilities...the Robinson-Eatwell analysis is hampered ... by their building the economic *priora quoad nos* of profits, wages, **prices, quantities** into explanation, when in fact the *priora quoad nos*^[1] are last in analysis: **they require explanation.** [McShane, 1980, 124]^[2]

Footnote 8:

1) The postulate of **invariance** that expressions of **principles and laws** are **invariant** under inertial transformations or under any transformations

2) The **postulate and principle of equivalence**: It is impossible to distinguish between gravitational acceleration and the acceleration of a frame of reference.

Accordingly, when the symbolic form of a mathematical expression is unchanged by a transformation, the meaning of the expression is unchanged. But a transformation is a shift from one spatio-temporal standpoint to another and, when expressions do not change their meaning under such shifts, then, ... the expressions are invariant and the ground of that invariance is that the expressions stand for abstract and generally valid propositions.

It follows that the mathematical expression of the principles and laws of a geometry will be invariant under the permissible transformations of that geometry. [CWL 3, 147/]

Footnote 7: Transformations

Such is the general principle, and it admits at least two applications. In the first application, one specifies successive sets of transformation equations, determines the mathematical expressions invariant under those transformations, and concludes that the successive sets of invariants represent the principles and laws of successive geometries.

In this fashion, one may differentiate Euclidean, affine, projective, and topological geometries. [CWL 3, 147/]

A second, slightly different application of the general principle occurs in the theory of Riemannian manifolds. The one basic law governing all such manifolds is given by the equation for the infinitesimal interval, namely,

$$ds^2 = \sum g_{ij} dx_i dx_j \quad [i, j = 1, 2, \dots, n]$$

where dx_1, dx_2, \dots are differentials of the coordinates, and where in general there are n^2 products under the summation. Since this equation defines the infinitesimal interval, it must be invariant under all permissible

Footnote 9: The principle of reciprocity

There is a sense in which one may speak of the fraction of basic outlay that moves to basic income as the "costs" of basic production. It is true that that sense is not at all an accountant's sense of costs; ... But however remote from the accountant's meaning of the term "costs," it remains that there is an aggregate and **functional** sense in which the fraction...

is an index of costs. For the greater the fraction that basic income is of total income (or total outlay), the less the remainder which constitutes the aggregate possibility of profit.

But what limits profit may be termed costs. Hence we propose ...to speak of ($c'O' = p'a'Q'$) and ($c''O'' = p''a''Q''$) as costs of production, having warned the reader that the costs in question are aggregate and **functional** costs.... [CWL 15, 156-57]

Footnote 10 Lonergan's commentary in CWL 15, 158-62 demonstrates the relativity of pricing and quantity, in a preto-quantality reminiscent of space-time, to one another through the phases of the pure cycle of expansion.