Edifice of Formulae of the Ideal Pure Cycle

For Review and Self-Test of Foundational and Superstructural Relations

(Upper-case letters represent flows of the form "so much or so many every so often")

One formula is worth a ton of prose.

Our purpose in this section is to remove the reader from the verbiage and allow him to concentrate solely on the relationships in the equations which explain the concrete economic process.[1] It is best if one understands the concise and precise mathematical forms, which are isomorphic[2] with the patterns of the dynamic process. One's understanding should be comprised of purely relational functionings.

Rebecca Goldstein noted that Kurt Gödel preferred to communicate in a technical manner so as to avoid pointless and combative conversations:

> Mathematical verbosity, as opposed to verbosity of any other sort, could not have better suited the personal eccentricities of Kurt Gödel, a man who had so much to say on the nature of mathematical truth and knowledge and certainty, but wanted to be able to say it using only the rigorous methodology of mathematics. With a proof in hand, he would not have to involve himself in the sorts of combative human conversations he regarded with distaste, maybe even with horror. Goldstein, Rebecca, *Incompleteness* (New York, NY: W.W. Norton & Company, 2005), 135

Equations are devoid of hot air and excess verbosity. In and of themselves, they are neither psychological nor political. They are purely relational forms. Equations offer us the advantages of a.) stating concisely and precisely at an adequate level of abstraction the implicit primary relationships among implicitly-defined explanatory terms, b.) suggesting that values for the secondary constants and variables be as determinate and precise as probabilities allow, and c) comprising a coherent, unified, and complete theory comprised of classical and statistical laws. A few equations of adequate generality may encapsulate tens, or even hundreds, of pages of verbiage.

Below are key "classical" equations upon which to focus for eventual mastery. In any conversations with those locked into the IS-LM, AD/AS, and Phillips Curve mentality, it may be wise to take a lesson from Gödel and simply ask which of Lonergan's generalizing and explanatory formulae they find to be mistaken.

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[1] Among these *Topics in Functional Macroeconomics* are four treatments which concentrate on the explanatory mathematical formulae, but present them in different orderings. The four treatments are entitled:

1. *Cluster of Key Relations: Seeing Production, Exchange, and Finance All in a Single View*
2. *Key Equations Module*
3. *Buildup of Formulae*
4. *The Formulation of Functional Interdependencies*

[2] See the section entitled Isomorphism (URL)

We have arranged this presentation of the classical laws of functional macroeconomic dynamics as an orderly progressing “upward” from basic terms and equations, which form a systematic core, to culminating differential and difference equations which explain the Why and How of the overall economic functioning. The view is arranged “from the bottom up” in 3 columns of Production, Monetary, and Unitary Equations. So, the reader should start at the bottom of the list, where there are a Goal, a note re Ideal velocities, a key symbol ***k***, and foundational formulas; then the reader can work his/her way “back up” to the top, where there is a culminating formula for Gross Domestic Functional Flows.

In this progression “up” through the formulae, the reader should have at hand the following diagrams from CWL 15.

- Figure 14-1, Diagram of Rates of Flow
- Figure 24-4, Growth of Surplus Production (*Q”*) over the Pure Cycle
- Figure 24-6, Growth of Basic Production (*Q’*) over a Pure Cycle
- Figure 24-7, Rate of Change of *dQ’/Q’* and *dQ”/Q”* over a pure cycle
- Figure 27-1 Rate of Change of *v*, *w*, and *f* during a pure cycle, Ideal Maximum *f*.

The reader can then “see” the equations “embedded” in these Figures. And one should thereby “see” the concrete creative-destructive process being properly “realized”. One would thus be using this as a self-test of one’s understanding of functional macroeconomic dynamics.[3]

Production	**Unitary**	**Monetary**
(Quantities subject to cyclic phases)	(Prices and Quantities)	(in initial consideration)

$$GDFF = P'Q' + \Pi''K'' = (p'a'Q'_{Basic}) + (p''a''Q''_{Ordinary\ surplus}) + (\pi''a''K''_{Purely\ expansionary}) + (\pi''a''K''_{R\&M})$$

Components of Gross Domestic Functional Flows in their fundamental explanatory form

$$\delta f = v\delta w + w\delta v$$ [4]

The differential of the behavior of the pure-surplus-income ratio

$$f = v\frac{I''}{I' + I''} = vw$$ [5]

The ratio of pure surplus income total income. Normatively rises and returns to zero.

$$d(\Pi''K''_{\text{Expansionary}}) = d(\pi''a''K''_{\text{Expansionary}})$$

Differentials giving expansion of investment

$$\Pi''K''_{\text{Surplus Receipts}} = \pi''a''K''_{\text{Expansionary Outlays}} + \pi''a''K''_{\text{Repair and Maintenance to Self}}$$ [6] $(= \Sigma F_i = vI'')$

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[3] As a separate exercise at another time, the reader should study a)

- Figure 29-1 Diagram of Superposed /circuits
- Figure 31-1 Diagram of Government Spending and Taxes,

so that he can understand and appreciate the possible superpositions of mitigative or distortive circulations of money passing non-normatively through the Redistributive Function, and b) the horizontal and vertical flowings from and to the Redistributive Function, so that he can understand proper and improper adjustments of the money supply so as to either enable the flows of an ordered process or torture the flows of a disordered process.

[4] CWL 15, 148-149

[5] CWL 15, 148

the functional correlation of pure surplus investment expenditures-receipts with expansionary investment outlays using quantity indices, selling-price indices, and profit-price indices (vectors).
Prices and quantities are defined implicitly by the interrelations within and between their functional flows rather than exogenously as a given.

$\pi''a''K''_{\textbf{Expansionary Outlays}} \rightarrow I''_{\text{Pure Surplus}} \rightarrow E''_{\text{Pure Surplus}} \rightarrow \Pi''K''_{\text{Pure Surplus}}$

$$\boldsymbol{\delta J = \delta a' + a''\delta R + R\delta a''}$$ [7]

The first-order differential equation of
the behavior of the basic-price-spread ratio
The ratio expands and contracts in accord with phases.

$$\frac{\boldsymbol{P'}}{\boldsymbol{p'}} = \boldsymbol{a'} + \boldsymbol{a''}\left(\frac{\boldsymbol{p''Q''}_{\boldsymbol{Ordinary\ Surplus}}}{\boldsymbol{p'Q'}_{\boldsymbol{Basic}}}\right)$$

$$\textbf{or}\quad \boldsymbol{J = a' + a''R}$$

To derive P'/p', or J, called "the basic price spread ratio"
Normatively ≥ 0

$$\boldsymbol{d(P'Q') = d(p'a'Q')_{\textbf{Basic}} + d(p''a''Q'')_{\textbf{Ordinary Surplus}}}$$

Differentials giving acceleration of expended incomes (P'Q') and "macroeconomic costs"

$$\boldsymbol{P'Q' = (p'a'Q')_{\textbf{Basic}} + (p''a''Q'')_{\textbf{R\&M or Ordinary Surplus}}}$$ [8]

The functional identification of basic-circuit velocitous expenditures (E'=P'Q') and macroeconomic costs (p'a'Q') and p"a"Q")
using quantity indices, selling-price indices, and profit-price indices (vectors).
Prices and quantities are defined implicitly by the interrelations within and between their functional flows rather than exogenously as a given.

$$\sum p_{ij}q_{ij} = \boldsymbol{P}\cdot\boldsymbol{Q} = PQ\cos A$$ [9]

Total revenue flows are the dot product of
price and quantity vector indices.

$\boldsymbol{P', \Pi'', p', p''_{Ordinary\ Surplus}, \pi''_{Pure}, \pi''_{Surplus\ R\&M}}$
Vector price indices

$\boldsymbol{Q', Q''_{Ordinary\ Surplus}, K''_{Pure}, K''_{Surplus\ R\&M}, a'Q', a''Q''_{Ordinary\ Surplus}, \alpha''K''_{Pure}, \alpha''K''_{Surplus\ R\&M}}$ [10]
Vector quantity Indices

Implicit definition of "costs" and "profits" [11]

There is a sense in which one may speak of the fraction of basic outlay
that moves to basic income as the "costs" of basic production. ...
For the greater the fraction that basic income is of total income (or total outlay),
the less the remainder which constitutes the aggregate possibility of profit.

$$\boldsymbol{r = i = \frac{k(k' - d' - 1) - s[(k' - d' - 1)l + dl']}{k + s[(k' - d' - 2) + (1 + d)l']}}$$

In Burley and Csapo's *Money Information in Lonergan-von Neumann Systems*[12]
the rate of flow in pure surplus income (*macroeconomic interest*)in their von Neumann model of

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6 CWL 15, 150

7 CWL 15, 160

8 CWL 15, 158

9 CWL 15, 74

10 CWL 15, 73 ff.

11 CWL 15, 156-57

12 [Burley and Csapo, 1992-1, 140]

the transition from one level of production to a higher level[13]

$$\boldsymbol{\alpha_2(=\beta_2) = 1 + \frac{(lk^N - l^N k)}{l'^N k} - d^N}\ ^{14}$$

In Burley's evolutionary model, for example, in a situation of full employment,
the growth factor $\boldsymbol{\alpha}_2$ (1+growth rate or 1+*r*) and the interest factor (1+interest rate or 1+*i*)
are defined in the characteristic equation of a matrix game
relating the coefficients of land and labor (l, l^N, l'^N), the productivity (k, k^N)
of old and new capital, and the depreciation (d^N) of new capital equipment.
This formulation is critical because it demonstrates the quantified interdependence
and mutual constraints among labor, capital, productivity, and depreciation.

$$\frac{dQ''}{Q''} > \text{ or } = \text{ or } < \frac{dQ'}{Q'}$$

Comparison of rate of change of quantities in the surplus
and basic circuits as a basis for determination of
phases of an expansion

$$I''^{\textit{Pure Surplus Income}} = \sum F_i = \sum (1 - w_i) n_i y_i\ ^{15}$$

Income expended for investment in all income strata

$$I'^{\textit{Basic Income}} = \Sigma w_i n_i y_i\ ^{16}$$

Income expended for consumer goods in all income strata

$$dI' = \Sigma(w_i dn_i + n_i dw_i + dn_i dw_i) y_i\ ^{17}$$

The equations specifying how to adjust of the rate
of saving to the requirements of the productive phase

$$\triangle M' = (S' - s'O') = \triangle T' + \triangle R' + (O' - R')^{18}$$

Formula in macroeconomic terms for the normative
Increase in the money supply for the basic circuit.
Normatively, new money should enter through the supply function.

$$\sum_{i'} \sum_0^{n-1} (f_{ij} + T_{ij}) = \sum_{i'} \sum_1^{n} (o_{ij} + t_{ij} - s_{ij})\ ^{19}$$

Formula in microeconomic terms of the circulating
money required in an interval of the economic process

$$\sum s_{ij} = \sum v_i r_{ij}\ ^{20}$$

Transitional payments cover prior contributors' initial payments.

$$G = c''O'' - i'O' = 0^{21}$$

The condition of equilibrium

[13] We need not here identify the constants and variables of the formula, since our only purpose is to demonstrate that the interest rate is a.) intrinsic to the process, and b.) interrelated to all the constants and variables of the process.

[14] [Burley 1992-2, 277]

[15] CWL 15, 134

[16] CWL 15, 134

[17] CWL 15, 134

[18] CWL 15, 67"Transfers to or from supply, (S' – s'O'), tend to equal the sum of the increments of aggregate turnover magnitudes in final payments (ΔR') and transitional payments (ΔT'). Of these, two, the increment in transitional payments will be the larger, since for each sale at the final market there commonly is a sale at a number of transitional markets." CWL 15, 67 The history of the development of money points to a preponderant role of increasing turnover magnitude in circuit accelerations. CWL 15, 68

[19] CWL 15, 66

[20] CWL 21, 169

[21] CWL 15, 54

$$\boldsymbol{Q = I}$$

A nation earns what it produces.
(Not an equation, because different units of measure; nevertheless, an identity)

$\mathbf{k_n[f'_n(t\text{-}a)\text{-}B_n] = f''_{n\text{-}1}(t) - A_{n\text{-}1}}$[22]
The lagged technical accelerator
allowing for slack and depreciation
in a Schumpeterian creative-destruction
economy

Foundational analytic distinctions[23]

There are levels of production; i.e. a productive hierarchy. Each level's applied factors have a distinct relation with elements in the emergent standard of living (SOL). Each level will have its own monetary circuit of outlays, incomes, expenditures, and receipts.
First analytic relation: Current-determinate-point-to-current-determinate-point (point-to-point) correspondence of productive factors with factor-elements in the emergent standard of living (SOL)
Second analytic relation: Current-determinate-point-to-indeterminate-future-series (point-to-line) correspondence of productive factors with factor-elements in future emergent SOLs
Also, point-to-surface; point-to-volume, etc.

$\boldsymbol{Q_i = \sum \sum Q_{ijk}}$[24]
Total flow of products is the sum
of all their factors of production.

$\boldsymbol{q_i = \sum \sum q_{ijk}}$[25]
Each product is a composition
of its component factors of production
(Not an equation, because different units of measure; nevertheless, an identity)

$\boldsymbol{p_i = \sum \sum p_{ijk}}$[26]
Each product has an accounting cost
equal to a double summation of the
pricing of its factors of production

$\boldsymbol{k}$[27]
The subscript k is a **rate** of application of a factor of production applied by a compensated human

d/dt or Δ/Δt
The economic process is a process of flows.
The basic terms are **velocities** of so much or so many
Instantaneously or of so much or so many per period

Goal: We seek the immanent intelligibility of the current purely dynamic processes of
a) production, exchange, and finance, and
b) orderly expansion to a higher level of activities.

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22 CWL 15, 37
23 CWL 15, 23-28
24 CWL 15, 30
25 CWL 15, 30
26 CWL 15, 30
27 CWL 15, 29-30